ESTIMATION OF PARAMETERS OF ZERO-ONE PROCESSES BY INTERVAL SAMPLING: AN ADAPTIVE STRATEGY

BY

M. BROWN, H. SOLOMON and M. A. STEPHENS

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DEPARTMENT OF STATISTICS
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1. INTRODUCTION

In a previous paper (Brown, Solomon and Stephens, hereafter, referred to as Part 1) the authors considered the problem of estimating the parameters of a zero-one double Poisson process when sampling was permitted only at equal intervals. Specifically, the time t in state zero is distributed with distribution $F(t) = 1-\exp(-\lambda t)$, $t>0$, and that in state 1 with distribution $F(t) = 1-\exp(-\mu t)$, $t>0$; sampling takes place at time intervals Δ, so that a sequence of zeros and ones is obtained, and it is required to estimate λ and μ. Two procedures were investigated. In Procedure 1 a fixed number of observations was taken; this has the drawback that one may see no change of state. In procedure 2 one observes till a fixed number of cycles is seen. Suppose the original observation is a zero; a cycle is a sequence of zeros followed by a sequence of ones, observed till a new zero shows that a new cycle is beginning.

For either procedure it is clear that properties of the maximum likelihood estimates, which were given for procedures 1 and 2, depend greatly on $\lambda^* = \lambda\Delta$ and $\mu^* = \mu\Delta$. Graphs were given, based on extensive Monte

Carlo studies, to illustrate this dependence. A line was drawn on the graph, giving the experimental values of $E(\lambda^*)$ against λ^*, and also lines one standard deviation on either side were drawn to enable confidence intervals to be calculated. This was done for graphs illustrating both Procedures 1 and 2, for various values of the ratio μ/λ.

Two conclusions from these graphs were the following:

(a) if the true λ^* (or μ^*) were any value greater than 1, the expectation of the estimate $\hat{\lambda}^*$ (or $\hat{\mu}^*$) would be approximately 1; thus working backwards, an apparent estimate near 1 gave no indication of the true value.

(b) reasonable confidence limits for λ^* could be found from $\hat{\lambda}^*$ only as $\hat{\lambda}^*$ became small, say roughly less than 0.4.

In this paper we investigate an adaptive strategy suggested by these two conclusions. We begin with a suitable Δ using Procedure 2, and, if estimates $\hat{\lambda}^*$ and $\hat{\mu}^*$ are too large, we successively halve Δ until they are both small enough. Then all the samples, obtained at each stage of this technique, are combined together to give estimates of λ and μ. This involves numerical solution of the maximum likelihood equations. We show, again from Monte Carlo studies, that for a reasonable number of cycles at each stage, not only is bias almost totally removed, but also estimates and confidence intervals can be found for much higher starting values (i.e. those at Stage 1) of λ^* and μ^* than for the one-sample strategy.

For convenience, we now recall the estimation formulas of Procedure 2. Suppose at, say, Stage 1, a sample of n cycles is taken, with sampling

interval Δ. Let p_{01} be the probability of event E that the state of the process changes from zero at the beginning of one interval to one at the end; p_{01} depends on λ, μ, and Δ. Let K_{01} be the number of events E observed in the data. Similarly let p_{00}, p_{10}, and p_{11} be defined, and also K_{00}, K_{10}, and K_{11}. Clearly $p_{00} = 1-p_{01}$ and $p_{11} = 1-p_{10}$. Finally, let X_i be the length of the i-th string of zeros, and let Y_i be the length of the i-th string of ones, and suppose $\overline{x} = \Sigma_i x_i/n$ and $\overline{y} = \Sigma_i y_i/n$.

Then (Lemma 2 of Part 1) estimates of p_{01} and p_{10} are $\hat{p}_{01} = 1/\overline{x}$ and $\hat{p}_{10} = 1/\overline{y}$. Suppose $S = \hat{p}_{01} + \hat{p}_{10}$. If $S \geq 1$, maximum likelihood estimates of $\lambda^* = \lambda\Delta$, and of $\mu^* = \mu\Delta$ do not exist. If $S < 1$, the estimates are

$$(1) \qquad \lambda^* = -\hat{p}_{01} \{\ln(1-S)\}/S$$

$$\mu^* = -\hat{p}_{10} \{\ln(1-S)\}/S$$

In the next section we set forth the proposed adaptive strategy.

2. AN ADAPTIVE STRATEGY

(a) If any estimates, say λ_0 and μ_0, exist of λ and μ, choose Δ to be Δ_1, so that $\lambda_0\Delta_1$ and $\mu_0\Delta_1$ are both less than a constant C. This constant will be decided by the narrowness of the confidence intervals required; the graphs of Part 1 suggest it should almost certainly be less than 0.4.

(b) Suppose $\lambda_1^* = \lambda\Delta_1$ and $\mu_1^* = \mu\Delta_1$. Sample from the process till n complete cycles have been observed (Stage 1) and estimate λ_1^* and μ_1^* by $\hat{\lambda}_1^*$ and $\hat{\mu}_1^*$, given by the formulae (1) above. If these are both below C, accept them. The graphs of Part 1 can then be used to improve the point estimates or to provide confidence intervals for λ_1^* and μ_1^*, and the estimates of λ and μ are given by dividing by Δ_1.

(c) If an estimate $\hat{\lambda}_1^*$ or $\hat{\mu}_1^*$ is too large, or if they do not exist, proceed to Stage 2. Let Δ be $\Delta_2 = \frac{1}{2}\Delta_1$, and take a second sample of n cycles. Use the Stage 2 sample only, to estimate $\lambda_2^* = \lambda\Delta_2$ and $\mu_2^* = \mu\Delta_2$, and if both are less than C, combine the Stage 1 and Stage 2 samples to obtain final estimates of λ and μ, in the manner to be shown below in Section (e).

(d) If λ_2^*, μ_2^* are <u>not</u> less than C, take a Stage 3 sample of n cycles, with interval $\Delta_3 = \frac{1}{2}\Delta_2$. Repeat this procedure, until the estimates, using the Stage m sample only, of $\lambda_m^* = \lambda\Delta_m$ and of $\mu_m^* = \mu\Delta_m$, (where $\Delta_m = \Delta_1/2^{m-1}$)

are both less than C. Then combine <u>all the samples</u>, from all stages, to obtain final estimates of λ and μ.

(e) Before proceding with later steps in the operating procedure we show how these final estimates λ and μ are to be found. To simplify the notation, let p_{01} be p and let p_{10} be r; then $p_{00} = 1-p$ and $p_{11} = 1-r$. Further, let K_{01}, K_{00}, K_{10}, K_{11} be k, l, v, w respectively. Finally, suppose for the moment $\Delta_1 = 1$, so $\Delta_j = 1/2^{j-1}$. At Stage j, the logarithm of the likelihood is

$L_j = k_j \ln p_j + l_j \ln (1-p_j) + v_j \ln r_j + w_j \ln (1-r_j)$ where the subscript j denotes the value for Stage j. The overall log-likelihood, for Stages 1 to m is

$$L = \sum_{j=1}^{m} L_j .$$

In these expressions, we have

$$p_j = \frac{\lambda}{\lambda+\mu} T_j \quad \text{and} \quad r_j = \frac{\mu}{\lambda+\mu} T_j$$

where $S_j = \exp \{-(\lambda+\mu) \ \Delta_j\}$, and $T_j = 1-S_j$.

We wish to maximize the log-likelihood; because of the changing Δ_j it is not now possible to work with the probabilities p_j and r_j, as was done in Lemmas 1 and 2 of Part 1, but we must find the derivatives of L with respect to λ and μ. These in turn require the derivatives of p_j and r_j. Let $\gamma = \lambda + \mu$; then it may be shown that

$$\frac{\partial p_j}{\partial \lambda} = \frac{\mu T_j}{\gamma 2} + \frac{\lambda S_j \Delta_j}{\gamma} \quad ;$$

$$\frac{\partial p}{\partial \mu} = \frac{-\lambda T_j}{\gamma^2} + \frac{\lambda S_j \Delta_j}{\gamma} \quad ;$$

$$\frac{\partial^2 p}{\partial \lambda^2} = \frac{-2\mu T_j}{\gamma^3} + \frac{2\mu S_j \Delta_j}{\gamma^2} + \frac{\lambda S_j \Delta_j^2}{\gamma}$$

$$\frac{\partial^2 p}{\partial \lambda \partial \mu} = \frac{(\lambda-\mu)T_j}{\gamma^3} - \frac{(\lambda-\mu)S_j \Delta_j}{\gamma^2} - \frac{\lambda S_j \Delta_j^2}{\gamma}$$

$$\frac{\partial^2 p}{\partial \mu^2} = \frac{2\lambda T_j}{\gamma^3} - \frac{2\lambda S_j \Delta_j}{\gamma^2} - \frac{\lambda S_j \Delta_j^2}{\gamma}$$

and corresponding expressions can easily be deduced for the derivatives of γ_j. Returning to the derivatives of L, we then have

$$\frac{\partial L}{\partial \lambda} = \sum_{j=1}^{m} \frac{\partial L_j}{\partial \lambda} .$$

$$= \sum_{j=1}^{m} \left\{ \left(\frac{k_j}{p_j} - \frac{l_j}{1-p_j} \frac{\partial p_j}{\partial \lambda} \right) + \left(\frac{v_j}{\gamma_j} - \frac{w_j}{1-r_j} \right) \frac{\partial r_j}{\partial \lambda} \right\}$$

and

$$\frac{\partial L}{\partial \mu} = \sum_{j=1}^{m} \frac{\partial L_j}{\partial \mu}$$

$$= \sum_{j=1}^{m} \left\{ \left(\frac{k_j}{p_j} - \frac{l_j}{1-p_j} \right) \frac{\partial p_j}{\partial \mu} + \left(\frac{v_j}{r_j} - \frac{w_j}{1-r_j} \right) \frac{\partial r_j}{\partial \mu} \right\} ,$$

and maximum likelihood estimates of λ and μ are given by

(2) $$\frac{\partial L}{\partial \lambda} = 0 \quad \text{and} \quad \frac{\partial L}{\partial \mu} = 0 .$$

These simultaneous equations can be solved by iteration, using as starting values the estimates of λ, μ obtained from the final stage m. The procedure is as follows, using Newton's method for two variables to generate the iteration. Suppose $\frac{\partial L}{\partial \lambda} = Z_1(\lambda,\mu)$ and $\frac{\partial L}{\partial \mu} = Z_2(\lambda,\mu)$ so that (2) becomes

(3) $$Z_1(\lambda;\mu) = 0 \ ; \quad Z_2(\lambda,\mu) = 0 .$$

We have

$$\frac{\partial Z_1}{\partial \lambda} = \sum_{j=1}^{m} \frac{\partial Z_{1j}}{\partial \lambda} , \quad \frac{\partial Z_1}{\partial \mu} = \sum_{j=1}^{m} \frac{\partial Z_{1j}}{\partial \mu} ,$$

where

$$\frac{\partial Z_{1j}}{\partial \lambda} = \left(\frac{-k_j}{p_j^2} - \frac{l_j}{(1-p_j)^2}\right)\left(\frac{\partial p_j}{\partial \lambda}\right)^2 + \left(\frac{k_j}{p_j} - \frac{l_j}{1-p_j}\right)\frac{\partial^2 p_j}{\partial \lambda^2}$$

$$+ \left(\frac{-v_j}{r_j^2} - \frac{w_j}{(1-r_j)^2}\right)\left(\frac{\partial r_j}{\partial \lambda}\right)^2 + \left(\frac{v_j}{r_j} - \frac{w_j}{1-r_j}\right)\frac{\partial^2 r_j}{\partial \lambda^2}$$

and

$$\frac{\partial Z_{1j}}{\partial \mu} = \left(\frac{k_j}{p_j} - \frac{l_j}{1-p_j}\right)\frac{\partial^2 p_j}{\partial \lambda \partial \mu} + \left(\frac{\partial p_j}{\partial \lambda}\right)\left(\frac{\partial p_j}{\partial \mu}\right)\left[\frac{-k_j}{p_j^2} - \frac{l_j}{(1-p_j)^2}\right]$$

$$+ \left(\frac{v_j}{r_j} - \frac{w_j}{1-r_j}\right)\frac{\partial^2 r_j}{\partial \lambda \partial \mu} + \left(\frac{\partial r_j}{\partial \lambda}\right)\left(\frac{\partial r_j}{\partial \mu}\right)\left[\frac{-v_j}{r_j^2} - \frac{w_j}{(1-r_j)^2}\right] .$$

Corresponding expressions can easily be found for the derivatives of Z_2, with respect to λ and μ. Then suppose λ_0 and μ_0 are estimates of λ and μ, and let $s = \lambda - \lambda_0$ and $t = \mu - \mu_0$. Approximations to s and t are given by

$$\hat{s} = \frac{Z_2 \frac{\partial Z_1}{\partial \mu} - Z_1 \frac{\partial Z_2}{\partial \mu}}{\frac{\partial Z_1}{\partial \lambda}\frac{\partial Z_2}{\partial \mu} - \frac{\partial Z_1}{\partial \mu}\frac{\partial Z_2}{\partial \lambda}}, \qquad \hat{t} = \frac{Z_2 \frac{\partial Z_1}{\partial \lambda} - Z_1 \frac{\partial Z_2}{\partial \lambda}}{\frac{\partial Z_1}{\partial \mu}\frac{\partial Z_2}{\partial \lambda} - \frac{\partial Z_1}{\partial \lambda}\frac{\partial Z_2}{\partial \mu}}, \tag{4}$$

where the Z_1, Z_2, and the partial derivatives are all evaluated at λ_0 and μ_0. Then new solutions of (3) are $\lambda = \lambda_0 + \hat{s}$, and $\mu = \mu_0 + \hat{t}$; these values now become λ_0 and μ_0, and the above process is repeated from equations (4), to give successive sets of solutions until two sets are close enough to be regarded as accurate. A FORTRAN computer program is available from one of us (M.A.S.) to perform these calculations. We now return to the steps in the basic strategy.

(f) A reasonable figure for n appears to be 5. If $n = 5$, use the estimates to decide roughly the ratio $\mu/\lambda = r$ say. Figures 1, 2, and 3 then give the graph of $\hat{\lambda}$ versus λ, for $r = 1$, 2 and 4; also lines are drawn one standard deviation on either side, from which confidence intervals for λ can be found. For other values of r, intervals must be found by interpolation. The graphs may also be used to find intervals for μ if $\hat{\mu}$ appears on the y-axis, otherwise the intervals for λ must be used, with the estimated multiplier $\hat{r}$.

Example 1. For example, suppose we start with $\Delta_1 = 1$, and after four stages $\hat{\lambda}_4$ and $\hat{\mu}_4$ are both less than C. Let the final estimates then be $\hat{\lambda} = 1.28$ and $\hat{\mu} = 0.96$. Now μ is approximately equal to λ, so from Figure 1, entering at $\hat{\lambda} = 1.28$ we have confidence interval $0.97 < \lambda < 1.76$, and entering at 0.96 we have $0.72 < \mu < 1.32$.

Example 2. Suppose now, again after 4 stages, the final estimates are $\hat{\lambda} = 1.28$ and $\hat{\mu} = 2.88$. Figure 2 will be used, since $\hat{\mu}$ is approximately $2\hat{\lambda}$. The confidence interval for λ is $0.95 < \lambda < 1.73$, hardly different from that in Example 1. Since 2.88 cannot be entered on the vertical axis, the confidence interval for μ can be found by using that for λ and multiplying by $\hat{r} = \hat{\mu}/\hat{\lambda} = 2.25$. The interval is then $2.14 < \mu < 3.89$.

These intervals are roughly 66% intervals, based on a normal distribution for $\hat{\lambda}$ and $\hat{\mu}$, and this is an approximation. Other procedures, to find a joint confidence zone for both parameters, will be discussed in Section 3, after the results of the Monte Carlo studies have been presented.

3 TESTING THE STRATEGY

The strategy was tested by choosing λ and μ, with ratios $r = \mu/\lambda = 1, 1.5, 2$ and 4, and then following the stages above for $C = 0.4$. The final estimates $\hat{\lambda}$ and $\hat{\mu}$ were recorded, the number of stages required, and the total number of observations n_0 needed to make an estimate. This was repeated for 1000 Monte Carlo samples or more, and the mean of the estimates, called $\hat{\lambda}$, is plotted against λ in the Figures. The two other lines are as described above, one standard deviation on either side of the mean line. The standard deviation is estimated from the Monte Carlo samples. Values of n_0 are included in the figures.

<u>Comments</u>: Throughout these comments we assume that $\hat{\lambda}$ is the smaller of the two estimates.

(a) The first remarkable result is that, for $n = 5$, $\hat{\lambda}$ is almost equal to λ over the whole range covered; it is biased upward, but not strongly so, as it was in the one-stage sampling of Part 1. This is true also for the estimate $\hat{\mu}$, even when μ is four times λ. Values of b_1 and b_2, the skewness and kurtosis parameters, were calculated for the $\hat{\lambda}$ and $\hat{\mu}$ estimates and show the distributions to be not normal, but somewhat skew.

(b) Figure 4 shows the estimate of $\sigma(\hat{\lambda})$ and of $\sigma(\hat{\mu})$, obtained from the Monte Carlo samples. The standard deviation of $\hat{\lambda}$ appears to be practically independent of $r = \mu/\lambda$, at least for the situations considered here $(n = 5,\ r = 1,2,4,\ 0 < \lambda < 2.5)$, and the points shown are those

for $\mu = \lambda$, i.e. $r = 1$. The standard deviations of $\hat{\mu}$ are shown for $r = 2$ and 4; those for $r = 1$ are, as expected, similar to those for λ. Some points are shown for double runs with the same λ; they give an indication of the variability of $\hat{\sigma}(\hat{\mu})$. The graphs suggest that $\sigma(\hat{\lambda})$ and $\sigma(\hat{\mu})$ are practically linear in λ, with a rather strange zero effect at low λ. The standard deviations of $\hat{\mu}$ are not quite r times those of $\hat{\lambda}$, presumably because of correlation between the estimates. The suggestion to use r to obtain confidence intervals for μ, when they cannot be directly obtained from the figures, may therefore be conservative.

(c) The graphs show that confidence intervals for λ and μ, even though approximate, can now be found for $\hat{\lambda}$ up to, say, 2.0, a much higher limit than was available with the one-stage sampling of Problem 1.

(d) It is clear that n, the number of cycles observed per stage, will influence the results. Figure 6 gives the graph of $E(\hat{\lambda})$ against λ for $n = 2$, $\mu = \lambda$. The estimates have a greater bias than in Figure 1. They are obtained for a smaller average number of observations, but the standard deviation is then greater than for $n = 5$. It would appear that $n = 5$ is a reasonable number of cycles per stage.

(e) The estimates of λ and μ are correlated, and estimates of the correlation ρ were found from the Monte Carlo studies. It also appears to be independent of r, and the curve of $\hat{\rho}$ against λ, given in Figure 5, is compiled from all results for $r = 1, 2,$ and 4, $(n = 5)$. We then examined whether this correlation could be brought into the inference procedures, in a way similar to its use with a bivariate normal distribution. Consider the statistic

$$Z = \left\{ \frac{(\lambda-\lambda)^2}{\sigma_\lambda^2} - 2\rho \frac{(\hat{\lambda}-\lambda)\ (\hat{\mu}-\mu)}{\sigma_\lambda \sigma_\mu} + \frac{(\hat{\mu}-\mu)^2}{\sigma_\mu^2} \right\} \Big/ (1-\rho^2) \quad ;$$

Although the marginal distributions of λ and μ are skew, it might be still true that Z will be approximately χ_2^2 distributed as it would be

if $\hat{\lambda}$ and $\hat{\mu}$ were bivariate-normally distributed with means λ, μ, variances σ_λ^2, σ_μ^2, and correlation ρ. This was examined in a second batch of Monte Carlo runs, using values of the variances and correlations derived from the first Monte Carlo samples. These are only estimates (but very good estimates; they are based on 1,000 samples, and then smoothed) and it was found that Z is well approximated by χ_2^2 over the ranges of λ and μ considered.

Tests of hypotheses Thus to test the hypothesis that $\lambda = \lambda_0$ and $\mu = \mu_0$, we find the estimates $\hat{\lambda}$ and $\hat{\mu}$ from the strategy; obtain σ_λ, σ_μ, and ρ from Figures 4 and 5, and calculate Z; reject H_0 if Z is **greater** than $\chi_2^2(\alpha)$, the appropriate upper tail significance point, for level α, of the χ_2^2 distribution. Note that this significance point is given by $-2 \ln \alpha$.

Confidence intervals If the true values of σ_λ, σ_μ and ρ could be inserted in Z, a confidence ellipse could be found for λ and μ, biased on the estimates $\hat{\lambda}$ and $\hat{\mu}$, by setting $Z = \chi_2^2(\alpha)$. In practice, of course, these parameters will not be known; the estimates $\hat{\lambda}$ and $\hat{\mu}$ could be used to estimate them, using Figures 4 and 5, but so many new stochastic elements will now be introduced that Z will no longer be χ_2^2 distributed.

Nevertheless, the success of the χ_2^2 approximation in the case of known variances and correlations suggests that perhaps a formula like Z

can be found, based only on $\hat{\lambda}$, $\hat{\mu}$, λ and μ, which will again be χ_2^2 distributed and which can be used to give a confidence zone. After considerable experimentation, the following model was determined:

(1) Given the estimates $\hat{\lambda}$ and $\hat{\mu}$ ($\hat{\lambda} < \hat{\mu}$; $\hat{\gamma} = \hat{\mu}/\hat{\lambda}$), replace σ_λ by $a\hat{\lambda}$, σ_μ by $a\hat{r}\hat{\lambda}$, ρ by $1-\exp(-b\hat{\lambda})$, and then calculate Z.

(2) Set $Z = \chi_2^2(\alpha)$ to obtain a $100(1-\alpha)\%$ confidence ellipse for λ and μ.

The forms of the expressions above were suggested by Figures 4 and 5; it remained to find values a and b by experimentation, to give Z an approximate χ_2^2 distribution. For $n = 5$ cycles, good results were obtained, for $\lambda < 1.0$ and for r up to 4, with $a = 0.4$ and $b = 2.5$. The distribution of Z was tested by testing whether $\exp(-Z/2)$ is uniform between 0 and 1. Note that the a and b are not the values which would give the "best fit" to Figures 4 and 5; they work because the various compensating errors in the model give a Z which has the required distribution.

(f) Final Comments: Figures 1 to 3, apart from showing the results of the Monte Carlo studies, are useful to provide immediate approximate confidence intervals for the parameters. Although the model for Z above was derived empirically, the successful approximation of $\exp(-Z/2)$ by a uniform random variable means that a confidence ellipse can be drawn, with reliable confidence level $100(1-\alpha)\%$ by setting $\exp(-Z/2) = \alpha$ for all values of α. Further, the ellipse is a confidence

zone for both parameters taken together, and it can easily be drawn by computer, when estimates $\hat{\lambda}$ and $\hat{\mu}$ have been found. In fact, if a and b can be provided for other values of n, over reasonable ranges of λ and μ, to give Z the χ^2_2 distribution, the technique of estimation and the provision of confidence zones will depend in a minimum fashion on the true parameters, and can essentially be fully automated. Preliminary investigation for $n = 10$ suggests that this is possible. It should be worthwhile to investigate the technique further, to see, for example, how n_0 (the number of observations required), and the size of the confidence ellipse, depends on n.

Reference

Brown, M., Solomon, H., and Stephens, M. A. (1975), Estimation of parameters of Zero-one processes by Interval Sampling, To appear (1977), Operations Research.

Figure 1. Graph of $\hat{\lambda}$ against λ, $n = 5$; $\mu = \lambda$.

n_0 = average number of observations to obtain an estimate

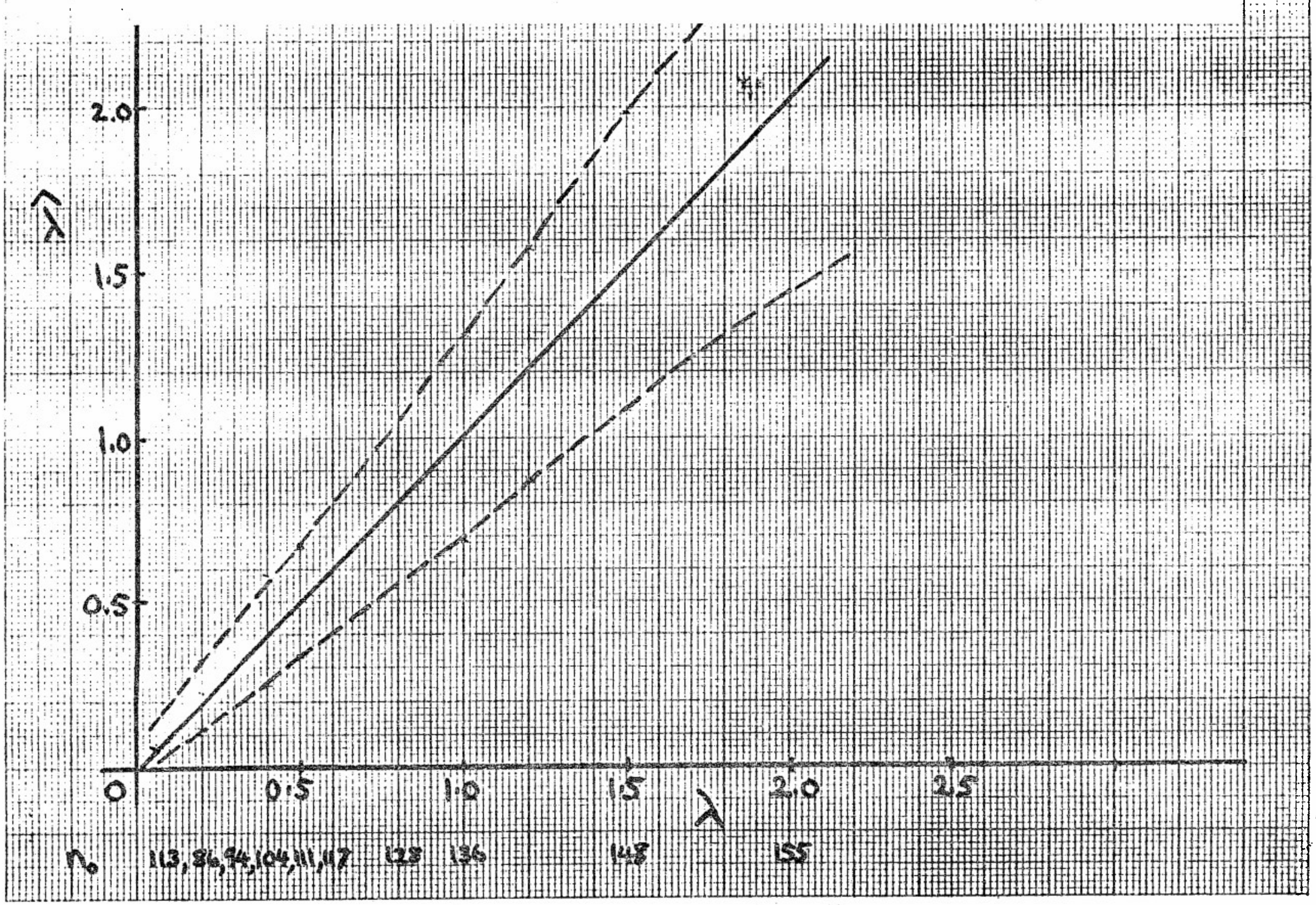


Figure 2. Graph of $\hat{\lambda}$ against λ, $n = 5$; $\mu = 2\lambda$.

n_0 = average number of observations to obtain an estimate

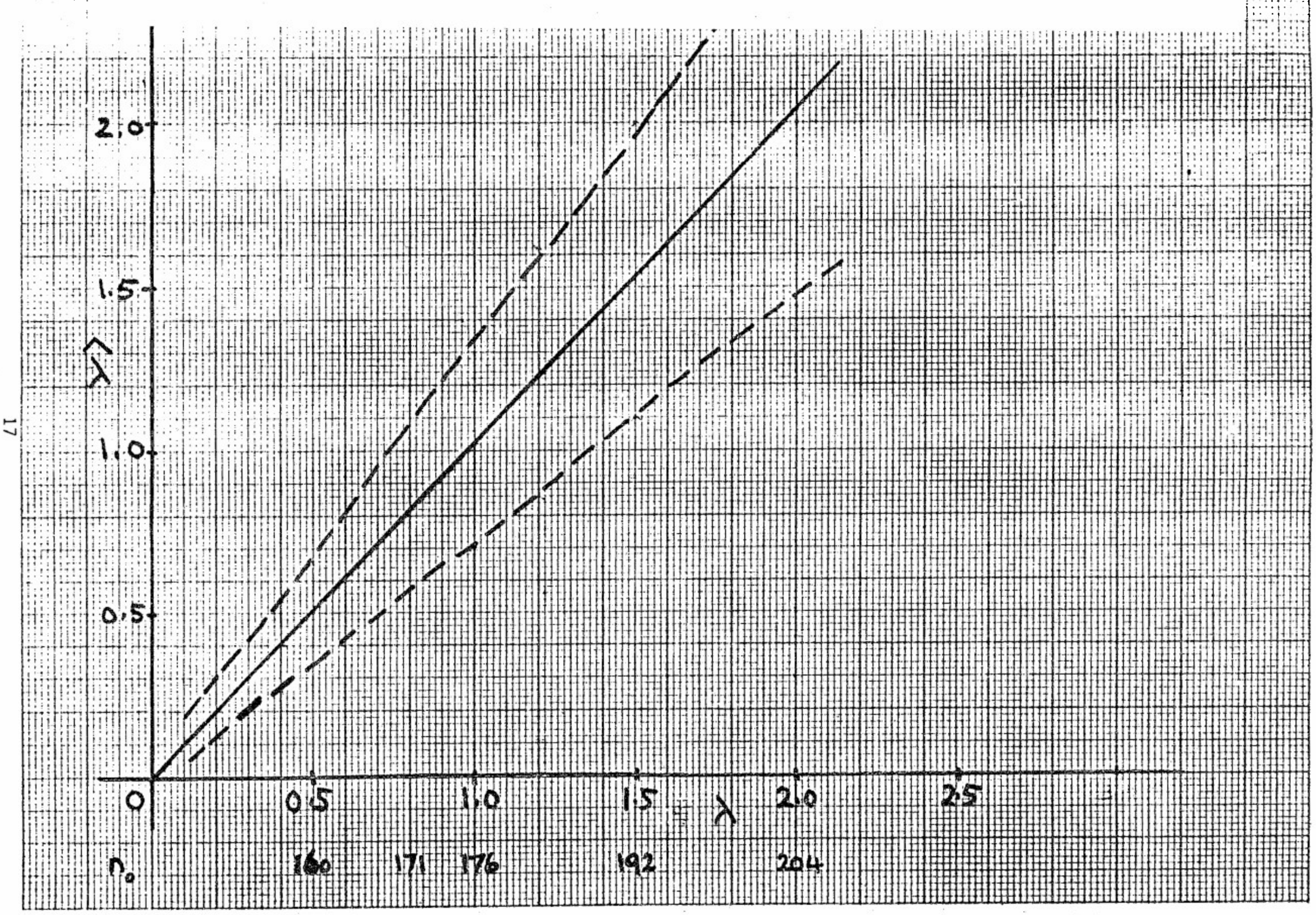


Figure 3. Graph of $\hat{\lambda}$ against λ , $n = 5$; $\mu = 4\lambda$.

n_0 = average number of observations to obtain an estimate

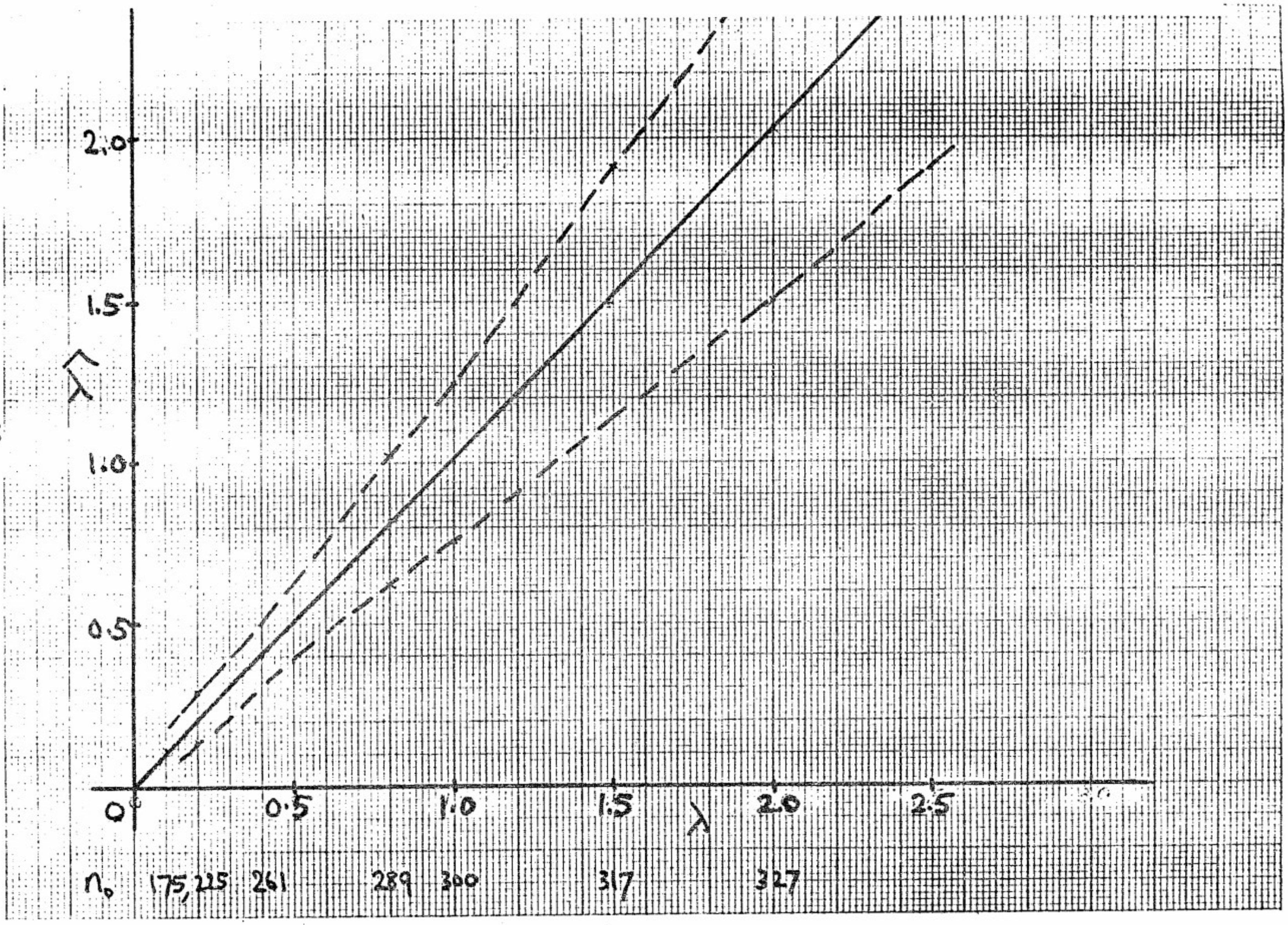


Figure 4. Graphs of $\hat{\sigma}(\hat{\lambda})$ and $\hat{\sigma}(\hat{\lambda})$, $n = 5$, $\mu = r\lambda$.

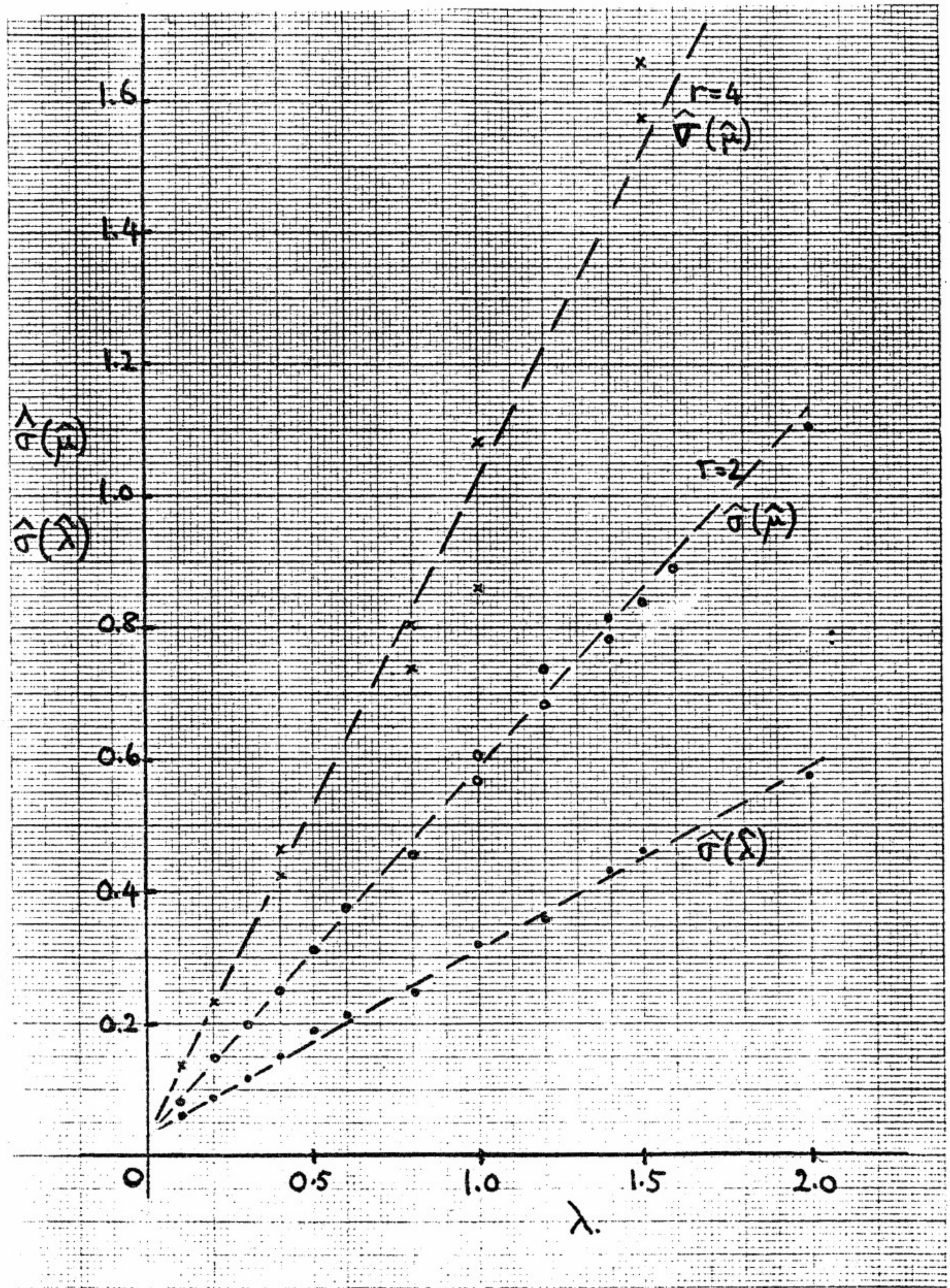


Figure 5. Graph of $\hat{\rho}$ against ρ , $n = 5$; $\mu = \lambda$.

n_0 = average number of observations to obtain an estimate

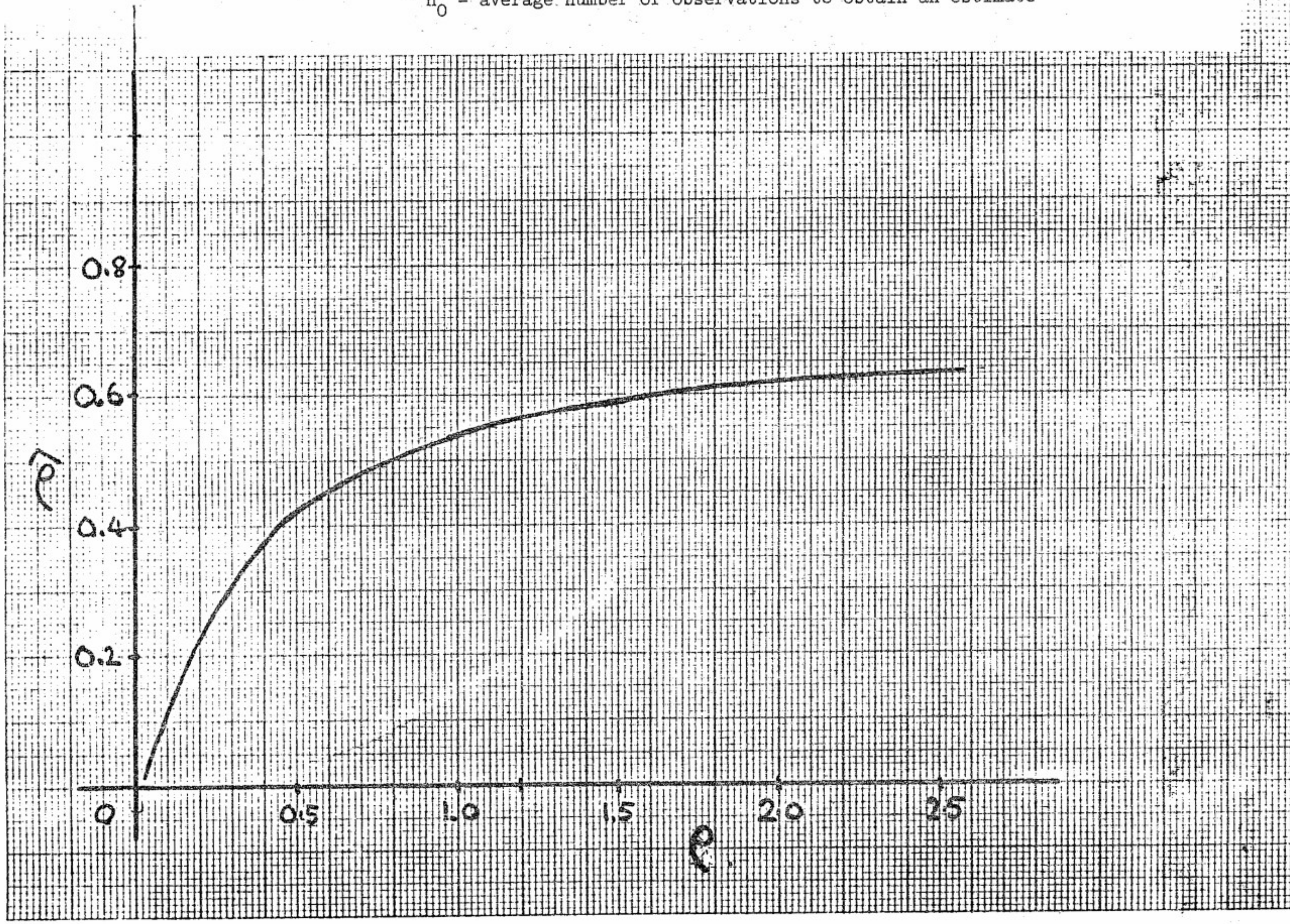
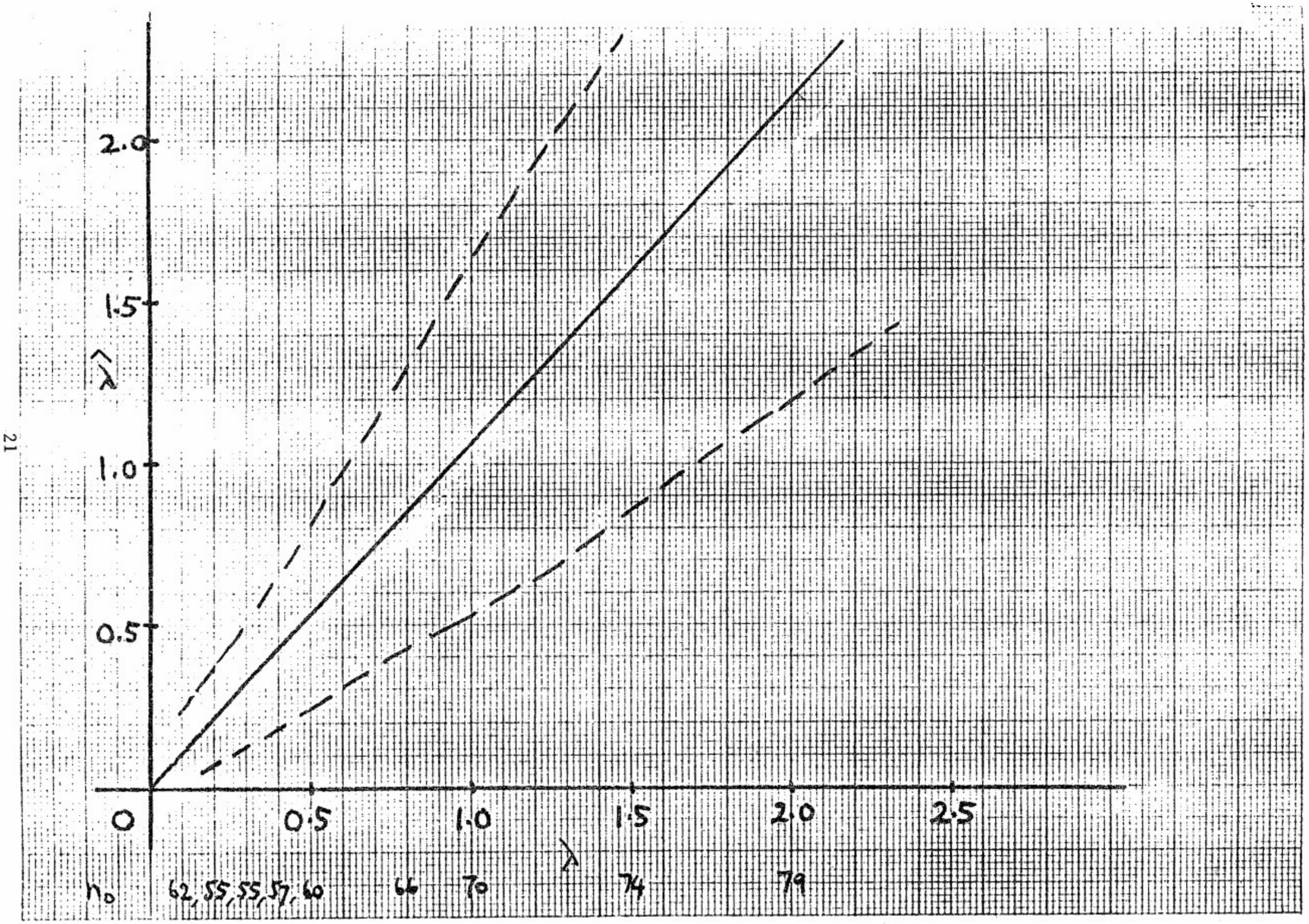


Figure 6. Graph of $\hat{\lambda}$ against λ , $n = 2$; $\mu = \lambda$.

n_{Ω} = average number of observations to obtain an estimate



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Abstract: In a previous paper, the authors considered the problem of estimating the parameters giving the mean time in each stage, for a two-stage Poisson process, when sampling was permitted only at equal intervals. It was impossible to get good results unless the intervals were small. We now propose an adaptive strategy in which the interval is successively halved until a suitable stage is reached; then all samples can be combined to give estimates. The strategy is examined by Monte Carlo methods, and it is shown to give a considerable improvement over the one-stage method. Figures are given to illustrate the results; they can be used also to improve estimates and give confidence intervals. A technique is proposed to give an approximate confidence ellipse for the two parameters, which appears to work well for the ranges considered.

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